

# Quantum Neural Network

- Optical Neural Networks operating at the Quantum Limit -

## Preface

We describe the basic concepts, operational principles and expected performance of a novel computing machine in this white paper. The first version of the machine was originally proposed in 2011 and named a coherent Ising machine. The network of injection-locked lasers represents Ising spin network, in which a coherent mean-field produced at above the oscillation threshold searches the ground state of an Ising Hamiltonian as a single lasing mode with a minimum overall loss. Later in 2013, the concept was extended to the network of degenerate optical parametric oscillators (DOPO), in which the intrinsic quantum uncertainty of the squeezed vacuum state, formed in each DOPO at below the oscillation threshold, searches the ground state of NP-hard Ising problems. The DOPO network based computing machine is called quantum neural network (QNN), which is distinct from classical neural network due to the following three properties:

1. Each DOPO is in a superposition state of different excitation amplitudes so that a quantum parallel search can be implemented.
2. A network of DOPOs makes a decision to reach a final computational result by correlated and collective symmetry breaking at a critical point of DOPO phase transition, i.e. oscillation threshold.
3. A network of DOPOs amplifies the above quantum solution obtained at a critical point to a classical signal via bosonic final state stimulation.

If a network of DOPOs is arranged into a recurrent neural network, such a device can find efficiently the satisfying solution of NP-complete  $k$ -SAT problems. Quantum noise is utilized as a useful computational resource and various quantum effects are employed to escape from wrong solutions (local minima) and chaotic traps in this case.

Various alternative approaches to modern digital computers have been intensively explored in recent years. There are four streams in the current efforts, which are

### 1. Return to analog computers

This approach promises a faster computer but suffers from intrinsic lack of precision due to noise injection and gate error.

### 2 Learn from nature

For example, a phase transition phenomenon at a critical temperature is considered as a super-efficient computational process, because such a system spontaneously realizes a single lowest-energy state out of huge number of possible states.

### 3 Mimic human brains

This approach covers a broad spectrum from commercial technology of deep machine learning chips to fundamental study on emergent mechanism for cognition and consciousness.

### 4 Utilize quantum effects

Quantum parallel search and quantum suppression of chaos are identified as two promising principles in quantum computers to exceed the limit of classical information processing, but they also suffer from intrinsic lack of precision just like analog computers.

QNN has all of the above four aspects so that it is hard to assign it to a specific category. As for the relation to category 1, the DOPOs operate as ideal analog memories which are robust against photon loss and noise injection. This fact allows QNN to solve not only a combinatorial optimization problem but also a continuous-variable optimization problem efficiently. As for the relation to category 2, the DOPOs make a decision to oscillate as either 0-phase coherent state or  $\pi$ -phase coherent state at a DOPO threshold (phase transition point). After this collective symmetry breaking (or supercritical pitchfork bifurcation) happens, bosonic final state stimulation is kicked in. The exponentially increasing success rate to find a ground state in QNN stems from the onset of this stimulated emission of photons at above threshold. As for the relation to category 3, the quantum dynamics in QNN resemble to the classical dynamics governed by the majority vote among many copies of classical neural networks (CNN) in human brain. A partial wave of the single quantum

wavefunction in QNN represents simultaneously many trajectories in CNN and therefore the final decision made by the single quantum wavefunction in QNN can be also based on the majority vote of many partial waves. As for the relation to category 4, quantum parallel search realized by squeezed vacuum states near the threshold provides an important step to solve an Ising problem and quantum suppression of classical chaos is a key to solve efficiently a  $k$ -SAT problem.

There are two types of quantum neural networks (QNN), i.e. optical delay line coupling based machine (DL-QNN) and measurement feedback coupling based machine (MF-QNN). These two machines use distinct quantum processes during the crucial preparation before the final decision making: quantum noise correlation (or entanglement) for DL-QNN and quantum wavepacket reduction for MF-QNN. The MF-QNN can implement not only symmetric neural network but also asymmetric recurrent neural network, which possesses a unique function of error detection and correction.

This white paper is organized as follows. Chapter I introduces the basic concepts, operational principles and expected performance of the DL-QNN and MF-QNN. The physics and nonlinear dynamics of degenerate optical parametric oscillators(DOPO) are presented in Chapter II, in which such topics as DOPO phase transition, quantum tunneling and effective temperatures are introduced. The quantum theory for the DL-QNN is presented in Chapter III, where quantum noise correlation and entanglement are identified as the important computational resource of the machine. Chapter IV briefly reviews the theory of quantum measurements, in particular the approximate measurements and continuous nonlinear measurements are formulated. The quantum theory of the MF-QNN is presented in Chapter V, where wavepacket reduction and contextuality are identified as the important computational mechanisms of this machine. Chapter VI describes the principles of the coherent Ising machines (CIM) based on numerical simulation results. The performance of CIM for NP-hard Ising problems is compared to the four types of classical neural networks: Hopfield network (discrete variables, deterministic evolution), simulated annealing (discrete variables, stochastic evolution), Hopfield-Tank neural network (continuous variables, deterministic evolution) and Langevin dynamics (continuous variables, stochastic evolution). Chapter VII describes the coherent SAT machines (CSM). The performance of CSM for NP-complete  $k$ -SAT problems is compared with the classical approach.

The readers interested in obtaining the minimum knowledge about the basic concepts and

principles of the QNN can start by reading Chapter I. If he/she is interested in the cloud service starting in November, 2017, Chapter VI provides a good summary for this novel computing machine. Finally, those who wish to understand the basic physics and quantum theory of the two types of QNN at a deeper level may read Chapter II - V as well as the above two chapters.

We will release several additional chapters for presenting coherent SAT machines and actual algorithms for real world problems: drug discovery, wireless communications, compressed sensing, deep machine learning and fintech in November, 2018.

The organization of the white paper are listed below:

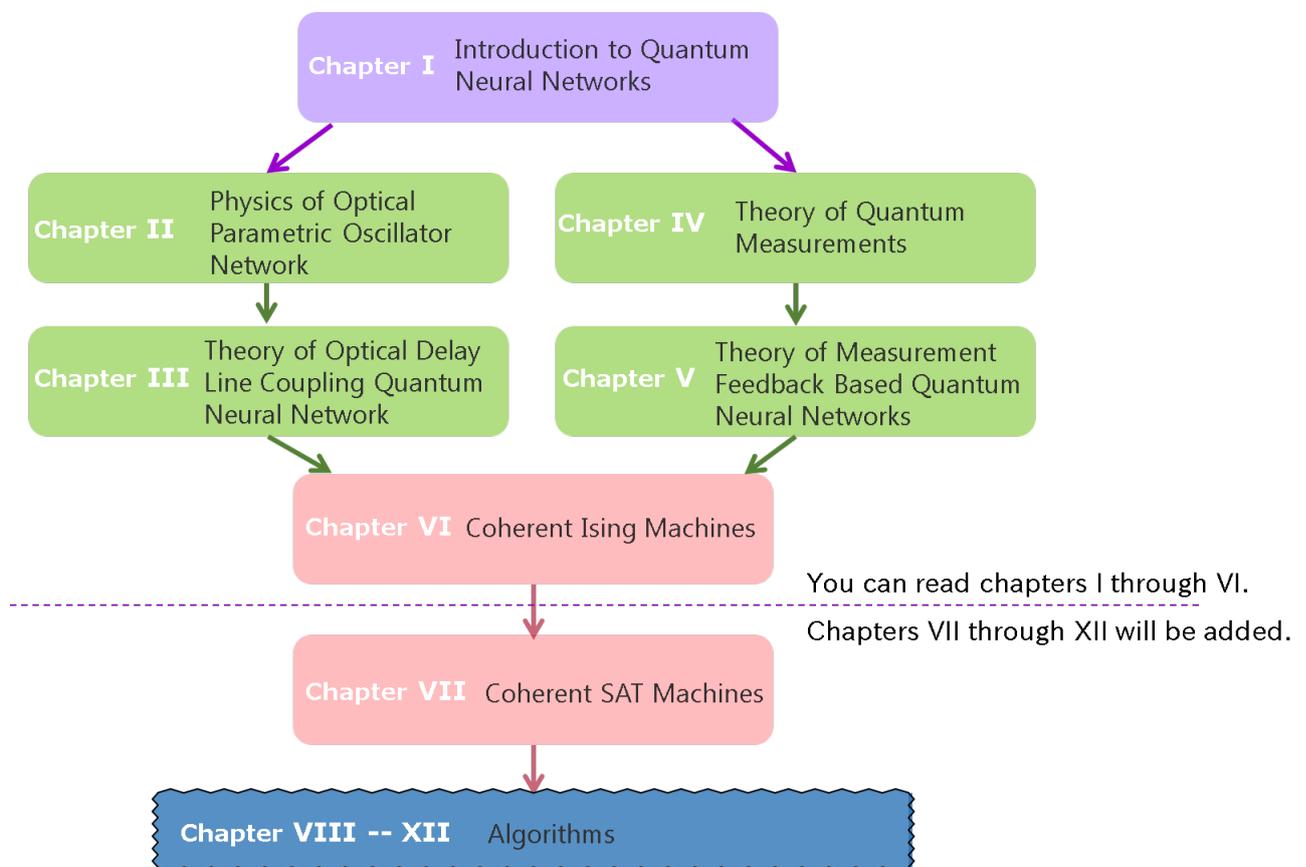


Figure 1: Organization of the white paper.

## I. Introduction to Quantum Neural Networks

### 1.1 Quantum neurons

1.1.1 Degenerate optical parametric amplifiers/oscillators

1.1.2 Linear superposition states in DOPA/DOPO

1.1.3 Amplitude and phase error correction by phase sensitive amplification

### 1.2 Quantum synapses

1.2.1 Optical delay line coupling scheme

1.2.2 Measurement feedback coupling scheme

### 1.3 Mapping of an Ising model to DOPO network: coherent Ising machines

1.3.1 Pitchfork bifurcation

1.3.2 Conditional mapping of the Ising Hamiltonian

1.3.3 Effect of the amplitude heterogeneity

### 1.4 Optical neural network at quantum limit and classical limit

### 1.5 Gottesman-Knill theorem

### 1.6 Summary

References

## II. Physics of Optical Parametric Oscillator Network

### 2.1 Parametric amplification

### 2.2 General quantum limit of linear amplifiers

### 2.3 Supercritical pitchfork bifurcation

### 2.4 OPO network

### 2.5 Example: Ising spins on cubic graph

### 2.6 Example: 1D Ising spin chains

2.6.1 Growth stage

2.6.2 Saturation stage

### 2.7 Correlation length and defect density

### 2.8 Multimode tunneling

2.8.1 Equations for signal fields and Hermite function expansion

2.8.2 Dynamics of the multimode tunneling

2.8.3 Simulated performance

### 2.9 XY machine based on nondegenerate OPO

2.9.1 Mechanism

2.9.2 1D chain

2.9.3 2D lattice

References

## III. Theory of Optical Delay Line Coupling Quantum Neural Network

### 3.1 Standard theoretical approach and computational difficulty

### 3.2 Positive $P(\alpha, \beta)$ representation

### 3.3 Truncated Wigner representation $W(\alpha)$

### 3.4 Quantum entanglement and inseparability

### 3.5 Quantum discord

### 3.6 Summary

References

## IV. Theory of Quantum Measurements

### 4.1 Exact measurements

### 4.2 Approximate measurements

#### 4.2.1 Measurement error and back action noise

#### 4.2.2 Measurement probability and post-measurement state

#### 4.2.3. Optical homodyne detection

### 4.3 Continuous measurements

### 4.4 Non-referred measurements

### 4.5 Linear and nonlinear continuous measurements

#### 4.5.1 Quantum Zeno effect

#### 4.5.2 Measurement-feedback QNN

### 4.6 Contextuality in quantum measurements

### 4.7 Summary

### References

## V. Theory of Measurement Feedback Based Quantum Neural Networks

### 5.1 A Quantum model based on density operator master equations and homodyne measurement projectors

#### 5.1.1 Theoretical formulation

#### 5.1.2 Numerical simulation results

### 5.2 A Quantum model based on $c$ -number stochastic differential equations and replicator dynamics

#### 5.2.1 Master equations

#### 5.2.2 Stochastic differential equations

#### 5.2.3 Numerical simulation results

### 5.3 Summary

### References

## VI. Coherent Ising Machines

### 6.1 Correlated spontaneous symmetry breaking in optical direct coupling QNN

### 6.2 Contextuality in measurement feedback QNN

### 6.3 Quantum parallel search, correlated symmetry breaking and bosonic final state stimulation

### 6.4 MAX-CUT problems

### 6.5 Coherent Ising machines vs. classical neural networks

#### 6.5.1 Coherent Ising machines (CIM)

#### 6.5.2 Classical neural networks

#### 6.5.3 Implementation of classical neural network algorithms

### 6.6 Simulation results

### 6.7 Discussion

#### 6.7.1 Validity for the hardware selection

#### 6.7.2 Optimization for PEZY-SC implementation for HTNN and CLD

### 6.8 Conclusion

### References